Engineering Notes

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Trajectory Perturbations of Asymmetric Fin-Stabilized Projectiles Caused by Muzzle Blast

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Nomenclature

A	$=\phi'_\infty/C$
B	$=\phi'_0/\phi'_\infty$
C	= roll damping coefficient
$J_A, J_{\xi'}$	= aerodynamic jump coefficients
s	= distance along trajectory nondimensionalized by projectile diameter
s_t	$=CS$
V_p	= projectile velocity
w	$=\mu+i\omega$, transverse velocity
δ,ϵ	= differential and asymmetric fin cant angles, respectively
ξ	$=\beta+i\alpha$, complex yaw angle
Θ	= angular deflection from boreline
ϕ	= roll angle
Φ_t	= roll-history integral defined in Eq. (4)

Subscripts

0	= properties immediately after muzzle blast
1	= properties at gun muzzle
∞	= free flight, steady-state properties

Superscripts

′	$=d/ds$

I. Introduction

AERODYNAMIC or inertial asymmetry of nonrolling fin-stabilized projectiles results in the round trimming to a fixed attitude once the launch disturbances damp out. To prevent continuously increasing divergence of the round from its intended trajectory, the projectile is given a small roll rate. Murphy and Bradley[1] have demonstrated that the aerodynamic jump due to asymmetries is dependent upon launch roll rate, steady-state roll rate, and the detailed roll rate history between these values. Since projectiles must traverse the gun muzzle blast prior to entry into free flight, the effective launch roll rate often differs from the in-bore value. In the reverse flow near the muzzle, the direction of the rolling moment due to fin cant is opposite to that in free flight; thus, the roll rate decreases from the in-bore value. For projectiles fired from smoothbore guns, a reverse spin may be produced.

In addition to modifying the projectile roll, muzzle blast imparts transverse linear and angular momentum to the round in direct relation to the magnitude of the asymmetry.

For asymmetric fin-stabilized projectiles, the angular trajectory deflection (Fig. 1) is found to be[1]

$$\Theta_0=w_0/V_p+J_{\xi'}\xi'_0+J_A\Phi_t/\phi'_\infty \tag{1}$$

The terms on the right-hand side of Eq. (1) are, respectively, deflection due to transverse velocity, aerodynamic jump due to transverse angular velocity, and aerodynamic jump due to asymmetry. The value ϕ'_∞ is the roll angular velocity in free flight.

In estimating the muzzle-blast effect upon the trajectory of an asymmetric 5.77 mm flechette, the present authors[2] assumed that the projectile is launched from a smoothbore gun with $w_l=\xi'_l=\phi'_l=0$ but with finite aerodynamic asymmetry. Their computations show that muzzle blast gasdynamic loadings induce transverse velocities and reverse spin, which in turn result in increased magnitudes of the individual terms in Eq. (1) over their values based on properties at the gun. However, vectorial summation of these terms shows the total jump to be equivalent to that predicted without consideration of muzzle blast perturbations (Fig. 2). The present paper discusses the results of an analysis[3] to generalize the applicability of this closure phenomenon.

II. Analysis

Consideration is given to a projectile launched from a smoothbore gun. The projectile is taken to have an even number of fins, and the asymmetry is defined as an inclination of two opposing fins at an angle in the same sense for both; thus, the differential fin cant angle is unchanged. It is of interest to examine the difference between the trajectory deflections computed with and without muzzle blast effects considered:

$$\Delta\Theta=\Theta_0-\Theta_l=w_0/V_p+J_{\xi'}\xi'_0+[J_A/\phi'_\infty]\times[\Phi_t(\phi'_\infty,\phi'_0)-\Phi_t(\phi'_\infty,0)] \tag{2}$$

where it is assumed that the projectile leaves the muzzle with zero roll rate and zero transverse linear and angular velocities. The first assumption restricts the analysis to launches from smoothbore guns. The next two assumptions do not affect the generality of the present analysis. Initial transverse velocities do not significantly affect muzzle blast loadings.[4] As a result, the contribution of these terms to the trajectory deflection would be of equal magnitude in both Θ_0 and Θ_l, thereby cancelling out in Eq. (2).

Use of the model[4] of loadings upon the projectile due to muzzle blast permits Eq. (2) to be expressed as

$$\Psi=\frac{\Delta\Theta}{(w_0/V_p)+J_{\xi'}\xi'_0}=1+C\frac{\Phi_t(\phi_\infty',\phi_0')-\Phi_t(\phi'_\infty,0)}{\phi'_0} \tag{3}$$

where Ψ is the ratio of the closure difference $\Delta\Theta$ to the jump due to transverse linear and angular velocities caused by muzzle blast. For a typical projectile, this jump is approximately 0.1 milliradian per degree of initial asymmetry.

Received July 20, 1977; revision received Oct. 13, 1977.
Index categories: LV/M Dynamics and Control; LV/M Trajectories and Tracking Systems.

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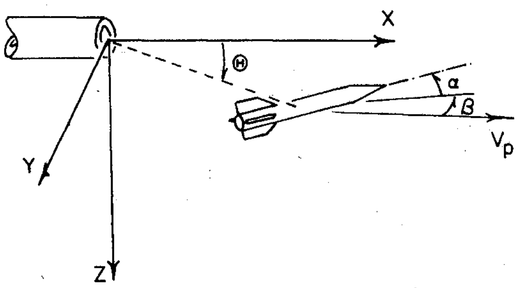

Fig. 1 Coordinate system for analysis.

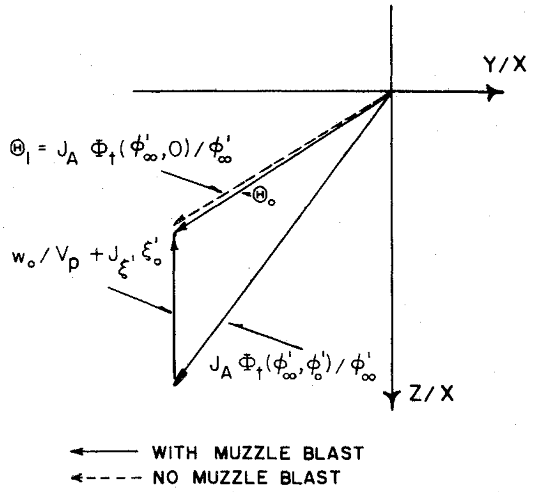

Fig. 2 Asymmetric jump with muzzle blast compared with asymmetric jump without muzzle blast.

In Eq. (3), Φ_t is the roll-history integral:[1]

$$\Phi_t = \lim_{s_t \to \infty} \frac{\phi'_\infty}{Cs_t} \int_0^{s_t}\int_0^{s_{t_2}} \exp(i\phi)\, ds_{t_1} ds_{t_2} \tag{4}$$

The roll angle ϕ is given as

$$\phi = (\phi'_\infty / C)\{s_t - [(\phi'_0/\phi'_\infty) - 1][\exp(-s_t) - 1]\} \tag{5}$$

Murphy and Bradley[1] have transformed the integral in Eq. (4) to

$$\Phi_t(A,B) = iA\int_0^\infty \exp[Af(r)]\,dr \tag{6}$$

where

$$f(r) = -\{r + i(B-1)[\exp(-ir) - 1]\} \tag{7}$$

In the present application, negative roll rates induced by the muzzle blast are considerably less than the free-flight roll rates; thus, the value of B will be small, permitting $\Phi_t(A,B)$ to be expanded in a Maclaurin Series:

$$\Phi_t(A,B) \approx \Phi_t(A,0) + \left.\frac{d\Phi_t}{dB}\right|_{B=0} B + \frac{1}{2}\left.\frac{d^2\Phi_t}{dB^2}\right|_{B=0} B^2 + \ldots \tag{8}$$

Differentiating Φ_t generates the factor $-iA[\exp(-ir)-1]$ that multiplies the original integrand. For large A and small B, the main contribution to the resulting integrals occurs for r small. If r is large, the integrands perform small oscillations about zero cancelling out in integration. For $B=0$, the integrand in Eq. (6) is expanded as

$$\exp[Af(r)] = [\exp(-iAr^2/2)]\exp(Au) \tag{9}$$

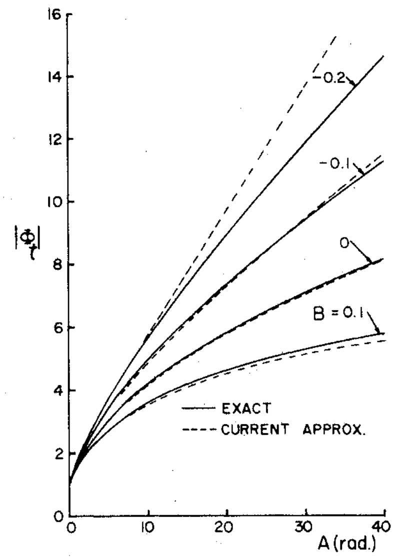

Fig. 3 Magnitude of Φ_t vs A.

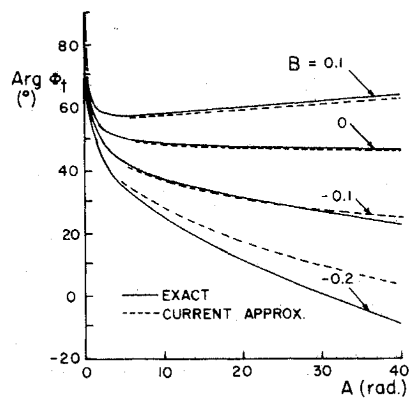

Fig. 4 Argument of Φ_t vs A.

where

$$u = -(r^3/6) + (ir^4/24) + (r^5/120) + \ldots \tag{10}$$

Similarly, the terms exp (Au) and $[\exp(-ir) - 1]$ are expanded in series for substitution into Eq. (6) and (8). By contour integration, the following forms of the integrals are obtained:

$$\int_0^\infty r^m \exp\frac{-iAr^2}{2}\,dr = \left(\frac{2}{A}\right)^{m/2} \frac{\exp[-i(m+1)\pi/4]}{2}\Gamma\left(\frac{m+1}{2}\right) \tag{11}$$

where Γ is the gamma function.

Evaluating the gamma functions, neglecting terms that have negative powers of A, and substituting into Eq. (8), provides

$$\Phi_t = \{[(1+i)(\pi A)^{1/2}/2] + [i/3]\} + [-A + (2i/3)]B + \{[(1-i)(\pi A)^{1/2}A/4] - [A/3] + [(1+i)(\pi A)^{1/2}/48] + [2i/135]\}B^2 + \ldots \tag{12}$$

In Figs. 3 and 4, this approximation for Φ_t is compared with the results of a Funge-Kutta integration of Eq. (4). The approximation is satisfactory up to $B = \pm 0.1$ but deteriorates for greater absolute values of B as A becomes large. For typical projectiles, $A \doteq 25$ and $B \doteq -0.05$; thus, the approximation is good over the region of interest.

Substitution of the lead terms of Eq. (12) into Eq. (3) yields

$$\Psi \doteq 1 + [1/AB]\{[-A + (2i/3)]B + [(1-i)(\pi A)^{1/2}A/4]B^2\}$$

$$\doteq [2i/(3A)] + [(1-i)(\pi A)^{1/2}B/4] \tag{13}$$

Since the lowest order terms cancel, $\Psi \doteq 0.1$, and the difference between the trajectory deflections computed including or neglecting muzzle blast effects is an order of magnitude less than the jump due to transverse linear and angular velocities; i.e., $\Delta\Theta - 0.01$ milliradians per degree of fin asymmetry.

III. Conclusions

An approximate analysis is used to obtain an expression for the effect of muzzle blast upon the jump of asymmetric fin-stabilized projectiles. While the muzzle blast increases the magnitudes of jumps due to transverse velocity and asymmetry, vector summation shows the two effects almost cancel. As a result, the jump can be adequately computed by completely neglecting the muzzle blast perturbation to the projectile dynamics.

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[2] Fansler, K.S. and Schmidt, E.M., "The Influence of Muzzle Gasdynamics Upon the Trajectory of Fin-Stabilized Projectiles," BRL R 1793, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Md., ADB005379L, June 1975.

[3] Fansler, K.S. and Schmidt, E.M., "Muzzle Blast Influence on Trajectory of Asymmetrical Fin-Stabilized Projectiles," BRL MR 2646, U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Md, AD B012784L, Aug. 1976.

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A. NO. OF ISSUES PUBLISHED ANNUALLY: 12
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